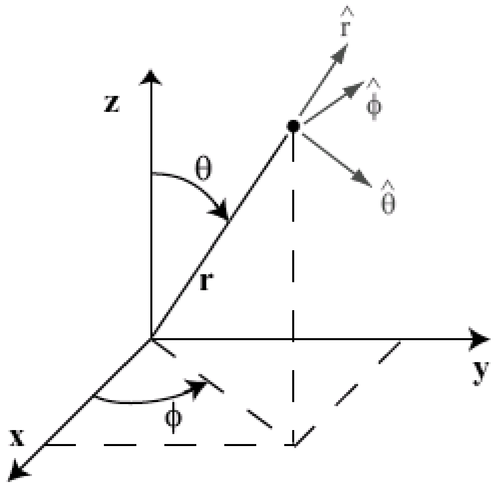
A Quick Note on Coordinate Conversions



A useful electric field expression is that of an electric dipole, which is two point charges *+q* and *–q* separated by distance *l*. If the two charges are located at the points *z=l/2* and *z=-l/2*, then the electric field is given approximately by $\vec{E} = \frac{ql}{4\pi\varepsilon_o r^3}\left(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta\right)$, which is valid far from the charges. To use this expression, we generally need to convert it to rectangular coordinates. To do this, we have to convert several different parts of the expression.

1) $r = \sqrt{x^2 + y^2 + z^2}$

2) $\sin\theta = \dfrac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$

3) $\cos\theta = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

4) $\hat{a}_r = \sin\theta\cos\phi\hat{a}_x + \sin\theta\sin\phi\hat{a}_y + \cos\theta\hat{a}_z$

5) $\hat{a}_\theta = \cos\theta\cos\phi\hat{a}_x + \cos\theta\sin\phi\hat{a}_y - \sin\theta\hat{a}_z$

6) $\cos\phi = \dfrac{x}{\sqrt{x^2 + y^2}}$

7) $\sin\phi = \dfrac{y}{\sqrt{x^2 + y^2}}$

Given these conversions, we have $\vec{E} = \frac{ql}{4\pi\varepsilon_o r^3}\left(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta\right)$, which is not very simple yet. Let us assume that we are to find the flux through a surface characterized by constant x. Then the flux integral requires us to find

$$\hat{a}_x \cdot \vec{E} = \frac{ql}{4\pi\varepsilon_o r^3}\left(2\cos\theta\hat{a}_x \cdot \hat{a}_r + \sin\theta\hat{a}_x \cdot \hat{a}_\theta\right) = \frac{ql}{4\pi\varepsilon_o r^3}\left(2\cos\theta\sin\theta\cos\phi + \sin\theta\cos\theta\cos\phi\right)$$

$$= \frac{ql}{4\pi\varepsilon_o r^3}\left(3\cos\theta\sin\theta\cos\phi\right) = \frac{ql}{4\pi\varepsilon_o r^3}\left(\frac{zx}{r^2}\right)$$

which is now much simpler to integrate over a given area in *y* and *z*.

As you can see, coordinate conversions are straight-forward but quite tedious.